



SECTION - A

1. This question contains 10 sub-parts each carrying ONE contains a multiple-choice question. Write in your answer number and the letter a, b, c or d corresponding to the most correct answer.
 - 1.1 The probability that it will rain today is 0.5. The probability that it will rain tomorrow is 0.6. The probability that it will rain either today or tomorrow is 0.7. What is the probability that it will rain today and tomorrow?

(a) 0.3 (b) 0.25 (c) 0.35 (d) 0.4
 - 1.2 The Newton-Raphson method is used to find the root of the equation $x^2 - 2 = 0$. If the iterations are started from -1 , the iterations will

(a) converge to -1 (b) converge to $\sqrt{2}$
 (c) converge to $-\sqrt{2}$ (d) not converge
 - 1.3 The determinant of the matrix

$$\begin{bmatrix} 6 & -8 & 1 & 1 \\ 0 & 2 & 4 & 6 \\ 0 & 0 & 4 & 8 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$
 is:

(a) 11 (b) -48 (c) 0 (d) -24
 - 1.4 The concatenation of two lists is to be performed on $O(1)$ time. Which of the following implementations of a list should be used?

(a) Singly linked list (b) Doubly linked list
 (c) Circular doubly linked list (d) Array implementation of list
 - 1.5 The correct matching for the following pairs is

(A) All pairs shortest paths	(1) Greedy
(B) Quick Sort	(2) Depth-First search
(C) Minimum weight spanning tree	(3) Dynamic Programming
(D) Connected Components	(4) Divide and Conquer

(a) A - 2 B - 4 C - 1 D - 3 (b) A - 3 B - 4 C - 1 D - 2
 (c) A - 3 B - 4 C - 2 D - 1 (d) A - 4 B - 1 C - 2 D - 3

1.6 In the following grammar

$$X ::= X \oplus \frac{Y}{Y}$$

$$Y ::= Z * \frac{Y}{Z}$$

$$Z ::= id$$

Which of the following is true?

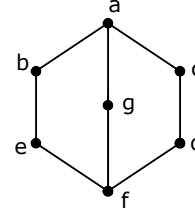
- (a) ' \oplus ' is left associative while '*' is right associative
 - (b) Both ' \oplus ' and '*' is left associative
 - (c) ' \oplus ' is right associative while '*' is left associative
 - (d) None of the above
- 1.7 Which of the following is essential for converting an infix expression to the postfix form efficiently?
- (a) An operator stack
 - (b) An operand stack
 - (c) An operand stack and an operator stack
 - (d) A parse tree
- 1.8 A language L allows declaration of arrays whose sizes are not known during compilation. It is required to make efficient use of memory. Which one of the following is true?
- (a) A compiler using static memory allocation can be written for L
 - (b) A compiler cannot be written for L; an interpreter must be used.
 - (c) A compiler using dynamic memory allocation can be written for L
 - (d) None of the above
- 1.9 The conditional expansion facility of macro processor is provided to
- (a) test a condition during the execution of the expanded program
 - (b) to expand certain model statements depending upon the value of a condition during the execution of the expanded program
 - (c) to implement recursion
 - (d) to expand certain model statements depending upon the value of a condition during the process of macro expansion.
- 1.10 Heap allocation is required for languages.
- (a) that support recursion
 - (b) that support dynamic data structures
 - (c) that use dynamic scope rules
 - (d) None of the above

2. The question contains 5 subparts, each carrying 1 mark. Each subpart contains a multiple-choice question. Write in your answer book the subpart number and the letter a, b, c or d corresponding to the most appropriate answer.
- 2.1 Let * be defined as $x*y = \bar{x} + y$. Let $z = x*y$. value of $z*x$ is
 (a) $\bar{x} + y$ (b) x (c) 0 (d) 1
- 2.2 RST 7.5 interrupt in 8085 microprocessor executes service routing from interrupt vector location
 (a) 0000H (b) 0075H (c) 003CH (d) 0034H
- 2.3 Purpose of a start bit in RS 232 serial communication protocol is
 (a) to synchronize receiver for receiving every byte
 (b) to synchronize receiver for receiving a sequence of bytes
 (c) a parity bit
 (d) to synchronize receiver for receiving the last byte
- 2.4 The correct matching for the following pairs is
 (A) DMA I/O (1) High speed RAM
 (B) Cache (2) Disk
 (C) Interrupt I/O (3) Printer
 (D) Condition Code Register (4) ALU
 (a) A - 4 B - 3 C - 1 D - 2 (b) A - 2 B - 1 C - 3 D - 4
 (c) A - 4 B - 3 C - 2 D - 1 (d) A - 2 B - 3 C - 4 D - 1
- 2.5 An N-bit carry lookahead adder, where N is a multiple of 4, employs ICs 74181 (4 bit ALU) and 74182 (4 bit carry lookahead generator).
 The minimum addition time using the best architecture for this adder is
 (a) proportional to N (b) proportional to $\log N$
 (c) a constant (d) None of the above
3. The question contains 10 subparts, each carrying 1 mark. Each subpart contains a multiple-choice question. Write in your answer book the subpart number and the letter a, b, c or d corresponding to the most appropriate answer.
- 3.1 Let $(Z, *)$ be an algebraic structure, where Z is the set of integers and the operation * is defined by $n*m = \text{maximum}(n, m)$. which of the following statements is true for $(Z, *)$?
 (a) $(Z, *)$ is a monoid (b) $(Z, *)$ is an Abelian group
 (c) $(Z, *)$ is a group (d) None of the above

- 3.2 Which of the following propositions is a tautology?
 (a) $(p \vee q) \rightarrow p$ (b) $p \vee (q \rightarrow p)$ (c) $p \vee (p \rightarrow q)$ (d) $p \rightarrow (p \rightarrow q)$

- 3.3 In the lattice defined by the Hasse diagram given in following figure, how many complements does the element 'e' have?

- (a) 2
 (b) 3
 (c) 0
 (d) 1



- 3.4 Given $\Sigma = \{a, b\}$, which one of the following sets is not countable?
 (a) Set of all strings over Σ
 (b) Set of all languages over Σ
 (c) Set of all regular languages over Σ
 (d) Set of all languages over Σ accepted by Turing machines

- 3.5 Locality of reference implies that the page reference being made by a process
 (a) will always be to the page used in the previous page reference
 (b) is likely to be to one of the pages used in the last few page references
 (c) will always be to one of the pages existing in memory
 (d) will always lead to a page fault

- 3.6 The correct matching for the following pairs is:

- | | |
|--------------------------|-----------------|
| (A) Disk scheduling | (1) Round robin |
| (B) Batch processing | (2) SCAN |
| (C) Time sharing | (3) LIFO |
| (D) Interrupt processing | (4) FIFO |

- | | |
|-----------------------------|-----------------------------|
| (a) A - 3 B - 4 C - 2 D - 1 | (b) A - 4 B - 3 C - 2 D - 1 |
| (c) A - 2 B - 4 C - 1 D - 3 | (d) A - 3 B - 4 C - 3 D - 2 |

- 3.7 I/O redirection
 (a) implies changing the name of a file
 (b) can be employed to use an existing file as input file for a program
 (c) implies connection 2 programs through a pipe
 (d) None of the above

- 3.8 When an interrupt occurs, an operating system
- ignores the interrupt
 - always changes state of interrupted process after processing the interrupt
 - always resumes execution of interrupted process after processing the interrupt
 - may change state of interrupted process to 'blocked' and schedule another process
- 3.9 Thrashing
- reduces page I/O
 - decreases the degree of multiprogramming
 - implies excessive page I/O
 - improve the system performance
- 3.10 Dirty bit for a page in a page table
- helps avoid unnecessary writes on a paging device
 - helps maintain LRU information
 - allows only read on a page
 - None of the above
4. The question contains 10 subparts, each carrying 2 marks. Each subpart contains a multiple-choice question. Write in your answer book the subpart number and the letter a, b, c or d corresponding to the most appropriate answer.
- 4.1 What is the maximum value of the function $f(x) = 2x^2 - 2x + 6$ in the interval $[0,2]$?
- (a) 6 (b) 10 (c) 12 (d) 5.5
- 4.2 Let $a = (a_{ij})$ be an n -rowed square matrix and I_{12} be the matrix obtained by interchanging the first and second rows of the n -rowed Identify matrix. Then AI_{12} is such that its first
- row is the same as its second row
 - row is the same as the second row of A
 - column is the same as the second column of A
 - row is all zero
- 4.3 Using a forward Euler method to solve $y''(t) = f(t)$, $y(0)$, $y'(0)=0$ with a step size of h , we obtain the following values of y in the first four iterations:
- $0, hf(0), h(f(0) + f(h))$ and $h(f(0) + f(h) + f(2h))$
 - $0, 0, h^2 f(0)$ and $2h^2 f(0)+f(h)$

- (c) $0, 0, h^2f(0) - 3h^2f'(0)$
 (d) $0, 0, hf(0) + h^2f'(0)$ and $hf'(0) + h^2f''(0) + hf'''(h)$
- 4.4 A polynomial $p(x)$ is such that $p(0) = 5$, $p(1) = 4$, $p(2) = 9$ and $p(3) = 20$. the minimum degree it can have is
 (a) 1 (b) 2 (c) 3 (d) 4
- 4.5 A binary search tree contains the value 1, 2, 3, 4, 5, 6, 7, 8. The tree is traversed in pre-order and the values are printed out. Which of the following sequences is a valid output?
 (a) 5 3 1 2 4 7 8 6 (b) 5 3 1 2 6 4 8 7
 (c) 5 3 2 4 1 6 7 8 (d) 5 3 1 2 4 7 6 8
- 4.6 Let $T(n)$ be the function defined by $T(1) = 1$, $T(n) = 2T\left(\lfloor \frac{n}{2} \rfloor\right) + \sqrt{n}$ for $n \geq 2$. Which of the following statements is true?
 (a) $T(n) = O(\sqrt{n})$ (b) $T(n) = O(n)$
 (c) $T(n) = O(\log n)$ (d) None of the above
- 4.7 A priority queue Q is used to implement a stack that stores characters. PUSH (C) is implemented as INSERT (Q, C, K) where K is an appropriate integer key chosen by the implementation. POP is implemented as DELETEMIN(Q). For a sequence of operations, the keys chosen are in
 (a) non-increasing order (b) non-decreasing order
 (c) strictly increasing order (d) strictly decreasing order
- 4.8 Given the following Pascal like program segment
- ```

Procedure A;
 x,y:integer;
 Procedure B;
 x,z:real
 S1
 end B;
 Procedure C;
 i:integer;
 S2
 end C;
end A;

```

The variables accessible in S1 and S2 are

- (a) x or A, y, x of B and z in S1 and x of B, y and i in S2
- (b) x or B, y and z in S1 and x of B, i and z in S2
- (c) x or B, z and y in S1 and x of A, i and y in S2
- (d) None of the above

4.9 The expression  $(a * b) * c$  op....

where 'op' is one of '+', '\*', and '^' (exponentiation) can be evaluated on a CPU with a single register without storing the value of  $(a * b)$  if

- (a) 'op' is '+' or '\*'
- (b) 'op' is '^' or '\*'
- (c) 'op' is '^' or '+'
- (d) not possible to evaluate without storing

4.10 The trapezoidal method to numerically obtain  $\int_a^b f(x) dx$  has an error E bounded by

$$\frac{b-a}{12} h^2 \max_{x \in [a,b]} f''(x)$$

where h is the width of the trapezoids. The minimum number of trapezoids guaranteed to ensure  $E \leq 10^{-4}$  is computing in 7 using  $f = \frac{1}{x}$  is

- (a) 60
- (b) 100
- (c) 600
- (d) 10000

5. The question contains 5 subparts, each carrying 2 marks. Each subpart contains a multiple-choice question. Write in your answer book the subpart number and the letter a, b, c or d corresponding to the most appropriate answer.

5.1 Let  $f(x, y, z) = \bar{x} + \bar{y}x + xz$  be a switching function. Which one of the following is valid?

- (a)  $\bar{y}x$  is a prime implicant of f
- (b)  $xz$  is a minterm of f
- (c)  $xz$  is an implicant of f
- (d) y is a prime implicant of f

5.2 Contents of A register after the execution of the following 8085 microprocessor program is

MVIA, 55 H  
MVI C, 25 H  
ADDC  
DAA

- (a) 7AH
- (b) 80H
- (c) 50H
- (d) 22H

- 5.3 A micro instruction is to be designed to specify
- (a) none or one of the three micro operations of one kind and
  - (b) none or upto six micro operations of another kind
- The minimum number of bits in the micro-instruction is
- (a) 9
  - (b) 5
  - (c) 8
  - (d) None of the above

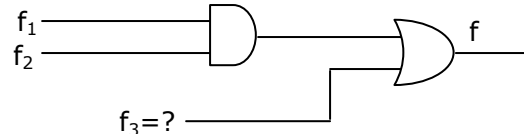
- 5.4 Given  $\sqrt{224}_r = 13)_r$ .
- The value of the radix r is:
- (a) 10
  - (b) 8
  - (c) 5
  - (d) 6

- 5.5 Consider the logic circuit shown in Figure below. The functions  $f_1, f_2$  and  $f$  (in canonical sum of products form in decimal notation) are:

$$f_1(w, x, y, z) = \Sigma 8, 9, 10$$

$$f_2(w, x, y, z) = \Sigma 7, 8, 12, 13, 14, 15$$

$$f(w, x, y, z) = \Sigma 8, 9$$



The function  $f_3$  is

- (a)  $\Sigma 9, 10$
- (b)  $\Sigma 9$
- (c)  $\Sigma 1, 8, 9$
- (d)  $\Sigma 8, 10, 15$

6. The question contains 10 subparts, each carrying 2 marks. Each subpart contains a multiple-choice question. Write in your answer book the subpart number and the letter a, b, c or d corresponding to the most appropriate answer.

- 6.1 A partial order  $\leq$  is defined on the set  $S = \{x, a_1, a_2, \dots, a_n, y\}$  as  $x \leq a_i$  for all  $i$  and  $a_i \leq y$  for all  $i$ , where  $n \geq 1$ . The number of total orders on the set  $S$  which contain the partial order  $\leq$  is

- (a)  $n!$
- (b)  $n+2$
- (c)  $n$
- (d) 1

- 6.2 Let  $G$  be a graph with 100 vertices numbered 1 to 100. Two vertices  $i$  and  $j$  are adjacent if  $|i - j| = 8$  or  $|i - j| = 12$ . The number of connected components in  $G$  is

- (a) 8
- (b) 4
- (c) 12
- (d) 25

- 6.3 The number of equivalence relations of the set  $\{1, 2, 3, 4\}$  is

- (a) 15
- (b) 16
- (c) 24
- (d) 4

- 6.4 Which one of the following regular expressions over  $\{0, 1\}$  denotes the set of all strings not containing 100 as a substring?

- (a)  $0^*(1+0)^*$
- (b)  $0^*1010^*$
- (c)  $0^*1^*01$
- (d)  $0(10+1)^*$



- 6.5 Which one of the following is not decidable?
- (a) Given a Turing machine  $M$ , a string  $s$  and an integer  $k$ ,  $M$  accepts  $s$  within  $k$  steps
  - (b) Equivalence of two given Turing machines
  - (c) Language accepted by a given finite state machine is not empty
  - (d) Language generated by a context free grammar is non empty

- 6.6 Which of the following language over  $\{a,b,c\}$  is accepted by a deterministic pushdown automata?

- (a)  $\{w \subset w^R \mid w \in \{a,b\}^*\}$
- (b)  $\{ww^R \mid w \in \{a,b,c\}^*\}$
- (c)  $\{a^n b^n c^n \mid n \geq 0\}$
- (d)  $\{w \mid w \text{ is a palindrome over } \{a,b,c\}\}$

Note:  $w^R$  is the string obtained by reversing 'w'.

- 6.7 An operating system contains 3 user processes each requiring 2 units of resource R. the minimum number of units of r such that no deadlocks will ever arise is

- (a) 3
- (b) 5
- (c) 4
- (d) 6

- 6.8 Each process  $P_i, i=1, \dots, 9$  is coded as follows

```
repeat
 P(mutex)
 {critical section}
 v(mutex)
forever
```

The code for  $P_{10}$  is identical except that it uses  $v(mutex)$  in place of  $P(mutex)$ . What is the largest number of processes that can be inside the critical section at any moment?

- (a) 1
- (b) 2
- (c) 3
- (d) None of the above

- 6.9 For a database relation  $R(a,b,c,d)$ , where the domains of  $a, b, c, d$  include only atomic values, only the following functional dependencies and those that can be inferred from them hold:

$a \rightarrow c$   
 $b \rightarrow d$

This relation is

- (a) in first normal form but not in second normal form
- (b) in second normal form but not in third normal form

- (c) in third normal form
- (d) None of the above

6.10 Let R (a, b, c) and S(d, e, f) be two relations in which d is the foreign key of S that refers to the primary key of R. Consider the following four operations R and S

- (a) Insert into R
- (b) Insert into S
- (c) Delete from R
- (d) Delete from S

Which of the following is true about the referential integrity constraint above?

- (a) None of (a), (b), (c) or (d) can cause its violation
- (b) All of (a), (b), (c) and (d) can cause its violation
- (c) Both (a) and (d) can cause its violation
- (d) Both (b) and (c) can cause its violation

7. A D flip-flop is to be connected to an 8085-microprocessor chip as a 1-bit output port with a port address of FF hex. Data bit D<sub>3</sub> should be involved in the data transfer from CPU to the flip-flop. The flip-flop should be cleared on power ON.

- (a) Using only one NAND gate (fan in of 10), one NOT gate and one D flip-flop, draw the required interface logic circuit (only the relevant signals should be shown)
- (b) Write a program to generate a squarewave on the output of the flip-flop. ON and OFF periods of the square wave should be 7 bus cycles each.

8. Let L = {a<sub>1</sub>, a<sub>2</sub>, ....., a<sub>n</sub>} n ≥ 0 be a list whose Pascal representation is

```
type list = record
 next: ↑ list; val : integer end
```

The following function returns a list in which a<sub>2i</sub> and a<sub>2i-1</sub>, 1 ≤ i ≤ ⌊ $\frac{n}{2}$ ⌋ are interchanged. Complete the function by filling the boxes. Write the line number and the content of the box in your answer sheet.

1. function change (p: ↑ list): ↑ list;
2. var q,t : ↑ list;
3. begin
4. if p = nil then change := p
5. else if p ↑ next = nil then change :=
6. else begin
7. q : p ↑ next;
8.  := q;
9. t : q ↑ next;
10.  := p;
11.  := change(t)
12. end
13. end

9. Consider a graph whose vertices are points in the plane with integer co-ordinates  $(x,y)$  such that  $1 \leq x \leq n$  and  $1 \leq y \leq n$ , where  $n \geq 2$  is an integer. Two vertices  $(x_1, y_1)$  and  $(x_2, y_2)$  are adjacent iff  $|x_1 - x_2| \leq 1$  and  $|y_1 - y_2| \leq 1$ . The weight of an edge  $\{(x_1, y_1), (x_2, y_2)\}$  is  $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
- (a) What is the weight of a minimum weight-spanning tree in this graph? Write only the answer without any explanations.
- (b) What is the weight of a maximum weight-spanning tree in this graph? Write only the answer without any explanations?

10. Consider the following program in Pseudo-Pascal syntax.

program what:

var z: integer

procedure recur (x):

begin if  $x \leq 40$  then

begin x:  $x + z$ ;

recur (x);

z :=  $x + 10$

end

end(\*recur\*)

begin (\*what\*)

z = 10;

recur(z);

writeln(z)

end

- (a) Suppose the parameter to the procedure 'recur' is passed by value.
- (i) What value is printed by the program?
- (ii) How many time is 'recur' called?
- (b) What value is printed by the program if the parameter is passed by reference?

11. Consider the grammar

$S \rightarrow bSc$

$S \rightarrow PQR$

$P \rightarrow bPc$

$P \rightarrow \epsilon$

$Q \rightarrow cQd$

$Q \rightarrow \epsilon$

$R \rightarrow dRe$

$R \rightarrow \epsilon$

Where S, P, Q, R are non-terminal symbols with S being the start symbol; b, c, d, e are terminal symbols and  $\epsilon$  is the empty string. This grammar generates strings of the form  $b^i, c^j, d^k, e^m$  for some  $i, j, k, m \leq 0$ .

- What is the condition on the values of  $i, j, k, m$ ?
- Find the smallest string that has two parse trees.

**SECTION - B**

Answer any TEN questions from this section. All questions carry equal marks.

- Consider a hash table with  $n$  buckets, where external (overflow) chaining is used to resolve collisions. The hash function is such that the probability that a key value is hashed to a particular bucket is  $\frac{1}{n}$ . The hash table is initially empty and  $K$  distinct values are inserted in the table.
  - What is the probability that bucket number 1 is empty after the  $K$ th insertion?
  - What is the probability that no collision has occurred in any of the  $K$  insertions?
  - What is the probability that the first collision occurs at the  $K$ th insertions?
- Let  $F$  be the set of one-to-one functions from the set  $\{1, 2, \dots, n\}$  to the set  $\{1, 2, \dots, m\}$  where  $m \geq n \geq 1$ .
  - How many functions are members of  $F$ ?
  - How many functions  $f$  in  $F$  satisfy the property  $f(i)=1$  for some  $i, 1 \leq i \leq n$ ?
  - How many functions  $f$  in  $F$  satisfy the property  $f(i) < f(j)$  for some  $1 \leq i < j \leq n$ ?
- Let  $R$  be a reflexive and transitive relation on a set  $A$ . Define a new relation  $E$  on  $A$  as
 
$$E = \{(a, b) \mid (a, b) \in R \text{ and } (b, a) \in R\}$$
  - Prove that  $E$  is an equivalence relation on  $A$ .
  - Define a relation  $\leq$  on the equivalence classes of  $E$  as  $E_1 \leq E_2$  is  $\exists a, b$  such that  $a \in E_1, b \in E_2$  and  $(a, b) \in R$ . Prove that  $\leq$  is a partial order.
- Consider the following function.
 

```
Function F(n,m:integer):integer;
begin
 If (n <=0) or (m <=0) then F:=1
 else
 F:F(n - 1,m) + F(n,m - 1);
 end;
```

- Use the recurrence relation  $\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$  to answer the following questions. Assume that  $n, m$  are positive integers. Write only the answers without any explanation.
- What is the value of  $F(n,2)$ ?
  - What is the value of  $F(n,m)$ ?
  - How many recursive calls are made to the function  $F$ , including the original call, when evaluating  $F(n,m)$ .
16. A size balanced binary tree is a binary tree in which for every node the difference between the number of nodes in the left and right subtree is at most 1. The distance of a node from the root is the length of the path from the root to the node. The height of a binary tree is the maximum distance of a leaf node from the root.
- Prove, by using induction on  $h$ , that a size-balance binary tree of height  $h$  contains at least  $2^h$  nodes.
  - In a size-balanced binary tree of height  $h \leq 1$ , how many nodes are at distance  $h - 1$  from the root? Write only the answer without any explanations.
17. An array  $A$  contains  $n \geq 1$  positive integers in the locations  $A[1], A[2], \dots, A[n]$ . the following program fragment prints the length of a shortest sequence of consecutive elements of  $A$ ,  $A[i], A[i+1], \dots, A[j]$  such that the sum of their values is  $\geq M$ , a given positive number. It prints 'n+1' if no such sequence exists. Complete the program by filling the boxes. In each case use the simplest possible expression. Write only the line number and the contents of the box.
- begin
  - $i := 1; j := 1;$
  - Sum :=  ;
  - min:=n; finish:= false;
  - While not finish do
  - If  then
  - if  $j = n$  then finish:=true.
  - else
  - begin
  - $j := j + 1;$
  - sum:=
  - end
  - else
  - begin
  - If  $(j - i) < \text{min}$  then  $\text{min} := j - 1;$
  - sum:=sum -  $A[i];$
  - $i := i + 1;$

- 18. end
- 19. writeln (min +1);
- 20. end.

18. Consider the following piece of 'C' code fragment that removes duplicates from an ordered list of integers.

```

struct node{
 int val;
 struct nod *next;
};
typedef struct node Node;
Node *remove-duplicates (Node*head, int *j)
{
 Node *t1, *t2;
 *j=0;
 t1 = head;
 if (t1 != NULL) t2 = t1 → next;
 else return head;
 *j = 1;
 if (t2 == NULL) return head;
 while t2 != NULL)
 {
 if (t1.val != t2.val) -----→ (S1)
 {
 (*j)++; t1 → next = t2; t1 = t2: -----→(S2)
 }
 t2 = t2 → next;
 }
 t1 → next =NULL;
 return head;
}

```

Assume the list contains n elements (n – 2) in the following questions.

- (a) How many times is the comparison in statements S1 made?
- (b) What is the minimum and the maximum number of times statements marked S2 get executed?
- (c) What is the significance of the value in the integer pointed to by j when the function completes?

19. A  $B^+$  - tree of order  $d$  is a tree in which each internal node has between  $d$  and  $2d$  key values. An internal node with  $M$  key values  $M + 1$  children. The root (if it is an internal node) has between  $1$  and  $2d$  key values. The distance of a node from the root is the length of the path from the root to the node. All leaves are at the same distance from the root. The height of the tree is the distance of a leaf from the root.
- What is the total number of key values in the internal nodes of a  $B^+$  - tree with  $l$  leaves ( $l \geq 2$ )?
  - What is the maximum number of internal nodes in a  $B^+$ -tree of order  $4$  with  $52$  leaves?
  - What is the minimum number of leaves in a  $B^+$ -tree of order  $d$  and height  $h$  ( $h \geq 1$ )?
20. Construct a finite state machine with minimum number of states, accepting all strings over  $(a,b)$  such that the number of  $a$ 's is divisible by two and the number of  $b$ 's is divisible by three.
21. Given that  $L$  is a language accepted by a finite state machine, show that  $L^P$  and  $L^R$  are also accepted by some finite state machines, where

$$L^P = \{s | ss' \in L \text{ some string, } s'\}.$$

$$L^R = \{s | s \text{ obtainable by reversing some string in } L\}.$$

22. A language  $L$  is a subset of Pascal with the following constructs:
- Expressions involving the operators '+' and '<' only
  - Assignment statements
  - 'while' statements and
  - Compound statements with the syntax 'begin...end'
- Give an unambiguous grammar of  $L$ .

23. The language  $L$ , defined by the following grammar, allows use of real or integer data in expressions and assignment statements.

$$\langle \text{assign-stmt} \rangle ::= \langle LHS \rangle := \langle E \rangle$$

$$\langle E \rangle ::= \langle E \rangle + \langle T \rangle | \langle T \rangle$$

$$\langle T \rangle ::= \langle T \rangle * \langle V \rangle | \langle V \rangle$$

$$\langle V \rangle ::= id | (\langle E \rangle)$$

$$(LHS) ::= id$$

It is required to convert expression and assignment strings of  $L$  into postfix strings that use the type-specific operations  $(+,i)$ ,  $(+,r)$ ,  $(*,i)$ ,  $(*,r)$ ,  $(:=,i)$  and  $(:=,r)$

Write a syntax directed translation scheme to convert expression and assignment strings into the post-fix form. You may assume that the name and type of a variable can be obtained by making the function calls' give type (id) and give-name (id)\* respectively.

24. Consider the following program fragment in Pascal:

```
Program Main;
 var X: integer;
 procedure A;
 var Y: integer;
 procedure B;
 var Z: integer;
 procedure C;
 var Z: integer;
 begin (*procedure C*)
 :
 end (*procedure C*)
begin (*procedure B*)
 :
 C; (*call to C*)
 A; (*call to A*)
 :
end (*procedure B*)
begin (* procedure A*)
 :
 B; (*call to B*)
 :
 end (*procedure A*)
begin (*Main*)
```

